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Strong and weak gravitational field in $R + \mu^4/R$ gravity

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Abstract We introduce a new approach for investigating the weak field limit of vacuum field equations in $f(R)$ gravity and we find the weak field limit of $f(R) = R + \mu^4/R$ gravity. Furthermore, we study the strong gravity regime in $R+\mu^4/R$ model of $f(R)$ gravity. We show the existence of strong gravitational field in vacuum for such model. We find out in the limit $\mu \to 0$, the weak field limit and the strong gravitational field can be regarded as a perturbed Schwarzschild metric.

Keywords Spherically symmetric solution. f(R) gravity. General relativity

1 Introductions

Observations on supernova type Ia (Riess et al. 1998; Perlmutter et al. 1999), cosmic microwave background (Spergel et al. 2003) and large scale structure (Tegmark et al. 2004), all indicate that the expansion of the universe is not proceeding as predicted by general relativity, if the universe is homogeneous, spatially flat, and filled with relativistic matter. An interesting approach to explain the positive acceleration of the universe is $f(R)$ theories of gravity which generalize the geometrical part of Hilbert-Einstein lagrangian (Capozziello 2002; Carroll et al. 2004, 2005; Clifton and Barrow 2005; Nojiri and Odintsov 2003; Sawicki and Hu 2007b; Evans et al 2008; Aghmohammadi et al. 2009). One of the initiative $f(R)$ models supposed to explain the positive acceleration of expanding universe has $f(R)$ action as $f(R) = R - \mu^4/R$ (Carroll et al. 2004). After proposing the $f(R) = R - \mu^4/R$ model, it was appeared this model suffer several problems. In the metric formalism, initially Dolgov and Kawasaki discovered the violent instability in the matter sector (Dolgov and Kawasaki 2003). The analysis of this instability generalized to arbitrary $f(R)$ models (Faraoni 2006; Sawicki and Hu 2007a) and it was shown than an $f(R)$ model is stable if $d^2f/dR^2 > 0$ and unstable if $d^2f/dR^2 < 0$. Thus we can deduce $R - \mu^4/R$ suffer the Dolgov-Kawasaki instability but this instability removes in the $R + \mu^4/R$ model, where $\mu^4 > 0$. Furthermore, one can see in the $R - \mu^4/R$ model the cosmology is inconsistent with observation when nonrelativistic matter is present. In fact there is no matter dominant era (Amendola et al. 2007a,b; Evans et al 2008). However, the recent study shows the standard epoch of matter domination can be obtained in the $R + \mu^4/R$ model (Evans et al 2008).

It is obvious that a viable theory of gravity must have the correct newtonian limit. Indeed a viable theory of $f(R)$ gravity must pass solar system tests. After the $R - \mu^4/R$ was suggested as the solution of cosmic-acceleration puzzle, it has been argued that this theory is inconsistent with solar system tests (Chiba 2003). This claim was based on the fact that metric $f(R)$ gravity is equivalent to $\omega = 0$ Brans-Dicke theory, while the observational constraint is $\omega > 40000$. But this is not quite the case and it is possible to investigate the spherical symmetric solutions of $f(R)$ gravity without invoking the equivalence of $f(R)$ gravity and scalar tensor theory (Clifton and Barrow 2005; Cembranos

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2006; Sawicki and Hu 2007b; Multamaki and Vilja 2006; Capozziello et al. 2008, 2009; Saaidi et al 2010; Capozziello et al. 2010). It has been shown that some $f(R)$ models accept the Schwarzschild-de Sitter space-time as a spherical symmetric solutions of field equation(Multamaki and Vilja 2006). Hence $R-\mu^4/R$ model has a Schwarzschild-de Sitter solution with constant curvature as $R=\sqrt{3\mu^4}$ where this is not the case in $R+\mu^4/R$ model.

In this paper we study the $R+\mu^4/R$ model of $f(R)$ gravity. We find the static spherically symmetric solution of vacuum field equation in both weak field limit and strong gravity regime, moreover, the weak field analysis can be expanded on $f(R)$ models of the form $f(R)=R+\epsilon h(R)$.

2 Weak field limit

In this section we investigate the weak field solution of vacuum field equation in $f(R)$ theories of gravity. We are interested in model of the form $f(R)=R+\epsilon h(R)$, with ϵ an adjustable small parameter. The motivation for discussing these models is that the nonlinear curvature terms that grow at low curvature can lead to the late time positive acceleration, but during the standard matter dominated epoch, where the curvature is assumed to be relatively high, could have a negligible effect.

The field equations for these models are

$$G_{\mu\nu}=-\epsilon\left[G_{\mu\nu}+g_{\mu\nu}\Box-\nabla_\mu\nabla_\nu+\frac{g_{\mu\nu}}{2}\times\left(R-\frac{h(R)}{\varphi(R)}\right)\right]\varphi(R)+kT_{\mu\nu}, \tag{1}$$

where $\varphi(R)=dh(R)/dR$. Contracting the field equation we obtain

$$R=\epsilon\left[R-\frac{2h(R)}{\varphi(R)}+3\Box\right]\varphi(R)-kT. \tag{2}$$

Where for the vacuum $T_{\mu\nu},T=0$. If $\epsilon=0$ the above equations reduce to Einstein equation. Hence we suppose $G_{\mu\nu}$ and R in the r.h.s of Eqs.(1,2) can be neglected for small values of ϵ. Furthermore if the condition $\lim_{R\to0}[h(R)/\varphi(R)]=0$ is satisfied we can neglect this term too. Neglecting these terms leads to the following equations

$$G_{\mu\nu}=-\epsilon\left[g_{\mu\nu}\Box-\nabla_\mu\nabla_\nu\right]\varphi(R), \tag{3}$$

and

$$R=\epsilon3\Box\varphi(R). \tag{4}$$

The analysis of spherically symmetric solution can be carried out using schwarzschild coordinate

$$ds^2=-A(r)dt^2+B(r)^{-1}dr^2+r^2d\Omega^2. \tag{5}$$

In the weak field limit approximation the metric deviates slightly from the Minkowski metric, so we can write

$$\begin{aligned}&A(r)=1+a(r),\\&B(r)=1+b(r),\\&\mid a\mid,\mid b\mid\ll1.\end{aligned} \tag{6}$$

When solving the field equations(3,4) we will keep only terms linear in the perturbations $a(r)$, $b(r)$. Hence equations (3,4) leads to

$$\begin{aligned}&\frac{a'}{r}+\frac{b}{r^2}=-\epsilon\frac{2}{r}\frac{d\varphi(R)}{dr}\\&\frac{b'}{r}+\frac{b}{r^2}=-\epsilon\nabla^2\varphi(R),\end{aligned} \tag{7}$$

and

$$R=3\epsilon\nabla^2\varphi(R). \tag{8}$$

where ($\prime$) indicates a derivation with respect to r.

2.1 $f(R)=R^{1+\epsilon}$

This model is considered in (Clifton and Barrow 2005). It is shown that this model has an exact spherically symmetric vacuum solution and regarding the general line-element in Eq.(5), it may be written as

$$\begin{aligned}A(r)&=r^{2\epsilon(1+2\epsilon)/(1-\epsilon)}+c\,r^{-(1-4\epsilon)/(1-\epsilon)},\\B(r)&=\frac{(1-\epsilon)^2}{(1-2\epsilon+4\epsilon^2)(1-2\epsilon-2\epsilon^2)}\\&\quad\times\left(1+c\,r^{-(1-2\epsilon+4\epsilon^2)/(1-\epsilon)}\right),\end{aligned}$$

where c is a constant. In the limit $\epsilon\to0$, these solutions become

$$\begin{aligned}ds^2&=-\left(1+2\epsilon\ln r+\frac{c}{r}\right)dt^2+\left(1+2\epsilon+\frac{c}{r}\right)^{-1}dr^2\\&\quad+r^2d\Omega^2.\end{aligned} \tag{9}$$

because we seek the weak field limit, in above equation we assume $c/r\ll1$.

Since we are interested in the limit $\epsilon\to0$, we may expand $f(R)=R^{1+\epsilon}$ around $\epsilon=0$. Then we have

$$\begin{aligned}f(R)&=R+\epsilon R\ln R,\\h(R)&=R\ln R,\\\varphi(R)&=1+\ln R.\end{aligned}$$

It is clear that $h(R)$ satisfies the condition

$$\lim_{R\to 0}\left[h(R)/\varphi(R)\right]=0.$$

Inserting $\varphi(R)$ in the trace equation (8), the Ricci scalar is obtained as

$$R=-\frac{6\epsilon}{r^2}. \tag{10}$$

Then we arrive at the solutions of Eq.(7)

$$a=\frac{c}{r}+2\epsilon\ln r, \qquad b=\frac{c}{r}+2\epsilon, \tag{11}$$

where c is a constant. We can see our solutions are in agreement with the exact solutions (9). Also one can check neglecting R, $G_{\mu\nu}$ and $h(R)/\varphi(R)$ in Eq.(1, 2) is reasonable.

2.2 $f(R)=R+\epsilon\ln R$

For this model $\varphi(R)=1/R$. Solving trace equation (8) and field equations (7) we obtain

$$R=\frac{\sqrt{6\epsilon}}{r}, \tag{12}$$

and

$$a=b=-\frac{2M}{r}-\sqrt{\frac{\epsilon}{6}}r. \tag{13}$$

where M is a constant. Therefore the space time metric for empty space in this model is

$$\begin{aligned} ds^2 &= -\left(1-\frac{2M}{r}-\sqrt{\frac{\epsilon}{6}}r\right)dt^2 \\ &\quad +\left(1-\frac{2M}{r}-\sqrt{\frac{\epsilon}{6}}r\right)^{-1}dr^2+r^2d\Omega^2. \end{aligned} \tag{14}$$

We can see, the generalized Newtonian potential is

$$\Phi_G=-\frac{M}{r}-\frac{1}{2}\sqrt{\frac{\epsilon}{6}}r. \tag{15}$$

This generalized gravitational potential has two terms. The first term is the standard Newtonian potential and the second term make a constant acceleration, $+\sqrt{\epsilon/24}$, which is independent of the mass of star. In (Saffari and Rahvar 2008) this metric is used to address the Pioneer's anomalous.

2.3 $f(R)=R\pm\mu^4/R$

Based on equivalence between $f(R)$ gravity and Brans-Dicke theory with $\omega=0$, it was argued that this theory is inconsistent with solar system tests (Chiba 2003). Indeed by this approach the Post-Newtonian parameter is found as $\gamma_{PPN}=1/2$ while the measurements indicate $\gamma_{PPN}=1+(2.1\pm 2.3)\times 10^{-5}$ (Bertotti et al. 2003). Also we must note that using equivalence between $f(R)$ gravity and scalar tensor gravity one can find models which are consistent with the solar system tests. This consistency can be made by giving the scalar a high mass or exploiting the so-called chameleon effect(Mota and Barrow 2004; Khoury and Weltman 2004; Capozziello and Tsujikawa 2008; Faulkner et al. 2007). However, when one is using equivalence between $f(R)$ gravity and scalar tensor gravity, the continuity of scalar field or its equivalent, the Ricci scalar, at the matter boundary is crucial condition which is not the case in Einstein gravity. But in this work we don't adopt the continuity of Ricci scalar for solving the field equations. Instead, we suppose that when μ tends to zero we arrive at the Einstein gravity. Thus we find a solution for $1/R$ model which is radically different from other solutions in (Erickcek et al. 2006; Chiba et al. 2007).

For this model we have

$$\begin{aligned} h(R) &= \pm 1/R, \\ \varphi(R) &= \mp 1/R^2, \end{aligned} \tag{16}$$

where $h(R)$ fulfills the condition $\lim_{R\to 0}\left[h(R)/\varphi(R)\right]=0$. Solving Eqs.(7,8) we obtain

$$\begin{aligned} &R=\mp 7\alpha\mu^{\frac{4}{3}}r^{-\frac{2}{3}}, \\ &\frac{\mu^4}{R^2}=\frac{1}{49\alpha^2}\mu^{\frac{4}{3}}r^{\frac{4}{3}}, \\ &a=-\frac{2M}{r}\pm\frac{3}{4}\alpha\mu^{\frac{4}{3}}r^{\frac{4}{3}}, \\ &b=-\frac{2M}{r}\pm\alpha\mu^{\frac{4}{3}}r^{\frac{4}{3}}. \end{aligned} \tag{17}$$

where $\alpha^3=4/147$ and M is a constant. Therefore the metric for space time is

$$\begin{aligned} ds^2 &= -\left(1-\frac{2M}{r}\pm\frac{3}{4}\alpha\mu^{\frac{4}{3}}r^{\frac{4}{3}}\right)dt^2 \\ &\quad +\left(1-\frac{2M}{r}\pm\alpha\mu^{\frac{4}{3}}r^{\frac{4}{3}}\right)^{-1}dr^2+r^2d\Omega^2. \end{aligned} \tag{18}$$

We can use the isotropic form, by introducing a new radius, ρ, which defined as

$$r=\rho\sqrt{1+\frac{2M}{\rho}\pm\frac{3}{4}\alpha\mu^{\frac{4}{3}}\rho^{\frac{4}{3}}},$$

and therefore the equivalent metric can be read as

$$ds^2 = -(1-\frac{2M}{\rho}\pm\frac{3}{4}\alpha\mu^{\frac{4}{3}}\rho^{\frac{4}{3}})dt^2$$

$$+ \quad (1+\frac{2M}{\rho}\pm\frac{3}{4}\alpha\mu^{\frac{4}{3}}\rho^{\frac{4}{3}})(d\rho^2+\rho^2 d\Omega^2).$$

From above metric one can see in the asymptotic behavior,$\mu\to 0$, $\gamma_{PPN}\simeq 1$ can be obtained.
From Eq.(17) it is obvious that in the limit $\mu\to 0$, μ^4/R^2 tends to zero, so there is not singularity in the field equations. Also one can check neglecting R, $G_{\mu\nu}$, and $h(R)/\varphi(R)$ in Eq.(1, 2) is reasonable.

2.4 Interior solution in the $f(R)=R+\mu^4/R$ model

In this section we discuss the interior gravitational field in the spherically symmetric case of static mass distribution in the $f(R)=R+\mu^4/R$ model where $\mu\to 0$. So we seek a spherically symmetric, static solution and we adopt the metric(5). In this model we may rewrite field equation (1) and trace equation (2) as

$$G^{\nu}_{\mu} \quad = \quad \left(\delta^{\nu}_{\mu}R+G^{\nu}_{\mu}+\delta^{\nu}_{\mu}\Box-\nabla_{\mu}\nabla^{\nu}\right)\frac{\mu^4}{R^2}+kT^{\nu}_{\mu}, \quad (19)$$

$$R \quad = \quad 3\left(R-\Box\right)\frac{\mu^4}{R^2}-kT. \quad (20)$$

From Eq.(2) it is obvious that as $\mu\to 0$, $R\to -kT$, so assuming $\mu^4\ll -kT$, in the r.h.s of Eq.(19) we may neglect those terms that contain μ^4/R^2. Thus field equations (19) reduce to Einstein equations hence we may write

$$G^{\nu}_{\mu}\simeq kT^{\nu}_{\mu}. \quad (21)$$

furthermore the conservation equation, $T^{\mu\nu}{}_{;\nu}=0$, leads to

$$p'=-\frac{A'}{2A}(p+\rho), \quad (22)$$

where p,ρ are pressure and density of matter. To obtain metric components (A,B), we use Eq.(22) and rr and tt components of Eq.(21)

$$G^{r}_{r}=\frac{A'}{A}\frac{B}{r}+\frac{B-1}{r^2}\simeq kp, \quad (23)$$

$$G^{r}_{r}=\frac{B'}{r}+\frac{B-1}{r^2}\simeq -k\rho c^2. \quad (24)$$

Solving Eq.(23) we may write

$$B=1-\frac{1}{r}kc^2\int_0^r\rho(x)x^2dx+\mathcal{O}\left(\frac{\mu^4}{k^2T^2}\right). \quad (25)$$

From continuity of the metric component B(r), on the boundary surface $r=r_0$ we find

$$\frac{kc^2}{r_0}\int_0^{r_0}\rho(x)x^2dx+\alpha\mu^{\frac{4}{3}}r_0{}^{\frac{4}{3}}+\mathcal{O}\left(\frac{\mu^4}{k^2T^2}\right)=\frac{2M}{r_0}, \quad (26)$$

where in the above equation we used the empty space solution Eq.(18). From the above equation we may determine the parameter M. It is seen that in the $\mu\to 0$ limit this constant reduces to the Schwarzschild radius. Furthermore, according to cosmological studies $\mu^2=10^{-52}m^{-2}$ (Carroll et al. 2004) so, regarding a typical solar system, in Eq.(26) we may neglect terms at order $\mathcal{O}\left(\frac{\mu^4}{k^2T^2}\right)$.

From equation (26) it is obvious that the physical interpretation of the parameter M differ from that of general relativity. Also from the above equation it is clear that in the $1/R$ gravity the external solution depends on the shape of matter distribution.

3 Strong Gravity Regime in $R+\mu^4/R$ Model

In this section we investigate the existence of strong gravitational field for $f(R)=R+\mu^4/R$ model of $f(R)$ gravity. We can rewrite the field equation (1) as

$$G^{\nu}_{\mu}\left(1-\frac{\mu^4}{R^2}\right)=-\frac{1}{3}\delta^{\nu}_{\mu}R-\nabla_{\mu}\nabla^{\nu}\left(\frac{\mu^4}{R^2}\right), \quad (27)$$

where we have used the trace equation

$$R=-3[R+\Box]\left(\mu^4/R^2\right). \quad (28)$$

In the above equation we have neglected the energy-momentum tensor of matter because we investigate the strong gravitational field around a spherically symmetric distribution of matter. Adopting the general spherically symmetric metric (5), we can rewrite the trace equation (28) and (rr),(tt) components of field equation (27) as

$$\begin{aligned} - \quad & \left[B\left(\frac{d^2}{dr^2}+\frac{2}{r}\frac{d}{dr}\right)+\frac{1}{2}\left(B'+\frac{BA'}{A}\right)\frac{d}{dr}+R\right] \\ \times \quad & \left(\mu^4/R^2\right)=\frac{R}{3}, \end{aligned} \quad (29a)$$

$$\begin{aligned} & \left(\frac{BA'}{rA}+\frac{B-1}{r^2}\right)\left(1-\mu^4/R^2\right) \\ + \quad & \left(B\frac{d^2}{dr^2}+\frac{B'}{2}\frac{d}{dr}\right)\left(\mu^4/R^2\right)=-\frac{R}{3}, \end{aligned} \quad (29b)$$

$$\begin{aligned} & \left(\frac{B'}{r}+\frac{B-1}{r^2}\right)\left(1-\mu^4/R^2\right) \\ + \quad & \frac{BA'}{2A}\frac{d}{dr}\left(\mu^4/R^2\right)=-\frac{R}{3}, \end{aligned} \quad (29c)$$

where$(\prime)$ denotes derivation with respect to the (r). In the previous section we showed, $(R+\mu^4/R)$ model

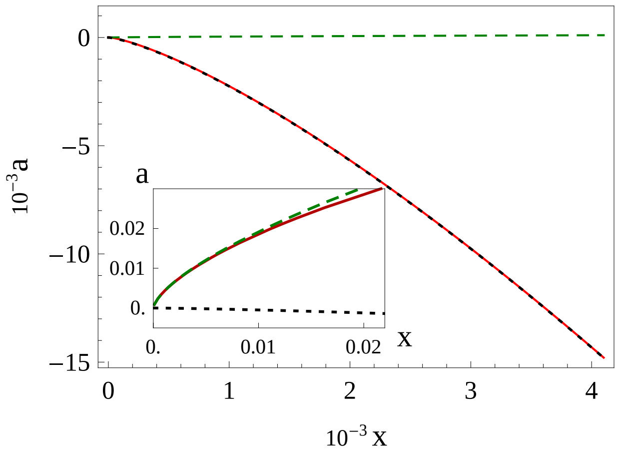

Fig. 1 a against x. The red-solid line shows numerical results of Eqs.(32). The green-dashed line represents approximate solution for $x \ll 1$ (Eq.(34a)) and the black-dotted line is the approximate solution for $x \gg 1$ (Eq.(36a)). A close up on the origin of main figure is presented .

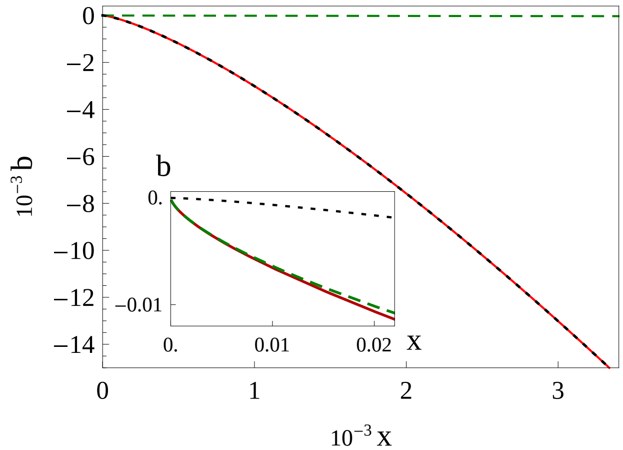

Fig. 2 b against x. The red-solid line shows numerical results of Eqs.(32). The green-dashed line represents approximate solution for $x \ll 1$ (Eq.(34b)) and the black-dotted line is the approximate solution for $x \gg 1$ (Eq.(36b)). A close up on the origin of main figure is presented .

has the week field solution as

$$\begin{aligned} ds^2 &= -\left[1 - \frac{2M}{r} + \frac{3}{4}\alpha(\mu r)^{\frac{4}{3}}\right] dt^2 \\ &+ \left[1 - \frac{2M}{r} + \alpha(\mu r)^{\frac{4}{3}}\right]^{-1} dr^2 + r^2 d\Omega^2, \end{aligned} \tag{30}$$

where $\alpha = (4/147)^{1/3}$. It is obvious this metric reduces to Schwarzschild metric in the limit $\mu \to 0$. Now we seek the solution of field equation in the limit $(r \to 2M)$. Without loss of generality we can assume $2M = 1$. In order to solve equations (29) we use some definitions as

$$\begin{aligned} &\phi = \gamma/R, \\ &\gamma = -\mu^{4/3}, \\ &A = 1 - \frac{1}{r} + \gamma a(r), \\ &B = 1 - \frac{1}{r} + \gamma b(r). \end{aligned} \tag{31}$$

Because we seek the solution in the limit $r \to 1$, we may define a new variable as $x = r - 1$. Using these definitions we can rewrite Eq.(29) as

$$\begin{aligned} &\gamma\left(b\frac{d}{dx} + \frac{2b}{x+1} + \frac{b'+a'}{2} + \frac{(x+1)(b-a)}{2(x+\gamma a(x+1))}\right. \\ &\left.\left(\frac{1}{(x+1)^2} + \gamma a'\right)\right)\frac{d\phi^2}{dx} = \left(\frac{1}{3} - \gamma\phi^2\right)\frac{1}{\phi} \\ &-\left(\frac{x}{x+1}\frac{d^2}{dx^2} + \frac{2x+1}{(x+1)^2}\frac{d}{dx}\right)\phi^2 \end{aligned} \tag{32a}$$

$$\begin{aligned} &\left(\frac{x}{x+1}\frac{d^2}{dx^2} + \frac{1}{2(x+1)^2}\frac{d}{dx} + \gamma\left(b\frac{d^2}{dx^2} + \frac{b'}{2}\frac{d}{dx}\right)\right)\phi^2 \\ &= \frac{1}{3\phi} + \left(\frac{b}{(x+1)^2} + \frac{a'}{x+1} + \frac{b-a}{x+\gamma a(x+1)}\right. \\ &\left.\times\left(\frac{1}{(x+1)^2} + \gamma a'\right)\right)(1+\gamma\phi^2) \end{aligned} \tag{32b}$$

$$\begin{aligned} &\frac{1}{2}\left(1 + \gamma\frac{(x+1)(b-a)}{x+\gamma a(x+1)}\right)\left(\frac{1}{(x+1)^2} + \gamma a'\right)\frac{d\phi^2}{dx} \\ &= \frac{1}{3\phi} + \left(\frac{b}{(x+1)^2} + \frac{b'}{x+1}\right)(1+\gamma\phi^2)\,, \end{aligned} \tag{32c}$$

where ($\prime$) denotes derivation with respect to the (x). For the limit $\mu \to 0$, in the above equations we suppose that we can neglect terms containing γ . After solving equations we check this assumption. By neglecting these terms, equations 32 can be rewritten as

$$\frac{1}{3\phi} = \left(\frac{x}{x+1}\frac{d^2}{dx^2} + \frac{2x+1}{(x+1)^2}\frac{d}{dx}\right)\phi^2 \tag{33a}$$

$$\begin{aligned} &\frac{b}{(x+1)^2} + \frac{a'}{x+1} + \frac{b-a}{x(x+1)^2} = -\frac{1}{3\phi} \\ &+\left(\frac{x}{x+1}\frac{d^2}{dx^2} + \frac{1}{2(x+1)^2}\frac{d}{dx}\right)\phi^2 \end{aligned} \tag{33b}$$

$$\frac{1}{2}\frac{1}{(x+1)^2}\frac{d\phi^2}{dx} = \frac{1}{3\phi} + \frac{b}{(x+1)^2} + \frac{b'}{x+1}. \tag{33c}$$

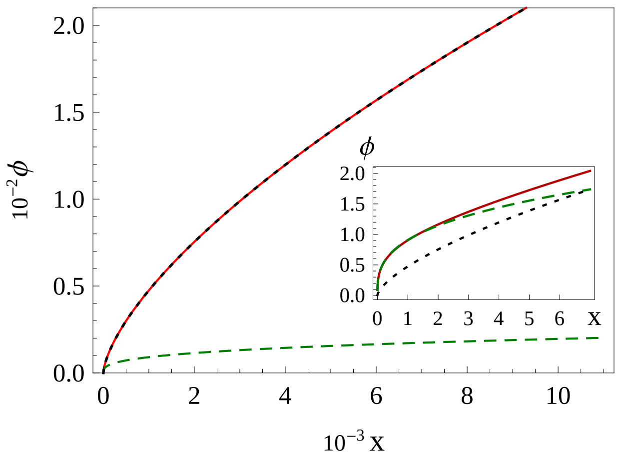

Fig. 3 φ versus x. The red-solid line shows numerical results of Eq.(32a). The green-dashed line represents approximate solution for $x \ll 1$ (Eq.(34c)) and the black-dotted line is the approximate solution for $x \gg 1$ (Eq.(36c)). A close up on the origin of main figure is presented .

In the limit $x \ll 1$, solutions of Eq. (33) are

$$a_0 = \frac{3}{8}\left(\frac{4}{3}\right)^{1/3} x^{2/3}, \tag{34a}$$

$$b_0 = -\frac{1}{8}\left(\frac{4}{3}\right)^{1/3} x^{2/3}, \tag{34b}$$

$$\phi_0 = \left(\frac{3}{4}\right)^{1/3} x^{1/3}. \tag{34c}$$

Thus we obtain the metric for $x \ll 1$ as

$$\begin{aligned} ds^2 &= -\left(1 - \frac{1}{r} - \frac{3}{8}\left(\frac{4}{3}\right)^{1/3} \mu^{4/3}(r-1)^{2/3}\right) dt^2 \\ &+ \left(1 - \frac{1}{r} + \frac{1}{8}\left(\frac{4}{3}\right)^{1/3} \mu^{4/3}(r-1)^{2/3}\right) dr^2 \\ &+ r^2 d\Omega^2. \end{aligned} \tag{35}$$

Furthermore, for $x \gg 1$, we can obtain the solutions of equations (33) as

$$a_\infty = -\frac{3}{4}\alpha x^{4/3}, \tag{36a}$$

$$b_\infty = -\alpha x^{4/3}, \tag{36b}$$

$$\phi_\infty = \frac{1}{7\alpha} x^{2/3}, \tag{36c}$$

which are in agreement with week field limit (30). Now we can check the validity of our assumption. Considering the solutions (36), shows that neglecting terms containing γ in Eqs. (32) is valid only for $x \gg\mid \gamma^3 \mid$ or $x \gg \mu^4$. Hence the metric (34) is solution of field equations in the range of $\mu^4 \ll x \ll 1$. By performing a conformal transformation and changing coordinate we can see the strong field solution (35) is

$$\begin{aligned} ds^2 = \\ -\left(1 - \frac{2M}{r} - \frac{3}{8}\left(\frac{4}{3}\right)^{1/3} (2M\mu)^{4/3}(\frac{r}{2M} - 1)^{2/3}\right) dt^2 \\ +\left(1 - \frac{2M}{r} + \frac{1}{8}\left(\frac{4}{3}\right)^{1/3} (2M\mu)^{4/3}(\frac{r}{2M} - 1)^{2/3}\right) dr^2 \\ +r^2 d\Omega^2, \end{aligned}$$

which is valid in the range of $(2M\mu)^4 \ll r/2M - 1 \ll 1$ and farther where $r \gg 2M$, the metric of space time can be approximated by the metric (30). Furthermore, we have solved field equations (32)numerically and presented the results in figures 1, 2, and 3. The plots show that the numerical results are in agreement with the analytical solutions (34,36) in their region of validity.

4 Discussion

We studied spherically symmetric solution of $f(R)$ gravity. At first a new approach for investigating the weak field limit of vacuum field equations in $f(R)$ gravity was introduced. Our results for the weak field limit of some studied f(R) models are in agreement with their known solutions. We solved the field equations for $f(R) = R + \mu^4/R$ gravity at weak field limit and obtained a solution which differs slightly from the schwarzschild metric. Our results are against the arguments that $f(R)$ models are ill defined because of the equivalence of $f(R)$ gravity and Brans-Dicke gravity with $\omega_{BD} = 0$ which leads to $\gamma_{PPN} = 1/2$. In fact our results are in agreement with the recent article of Capozziello et al. (Capozziello et al. 2010), in which they have studied Newtonian limit of the $f(R)$ gravity by considering that fourth order gravity models are dynamically equivalent to the O'Hanlon lagrangian and they have shown fourth order gravity models can not be ruled out only on the base of analogy with Brans-Dicke gravity with $\omega_{BD} = 0$. Moreover, regarding the results for the weak field limit, we investigated the strong field regime for this model and showed that if $(r - 2M)/(2M)^5 \gg \mu^4$, where r and $2M$ are radius and Schwarzschild radius in the Schwarzschild coordinate respectively, the gravitational field is a perturbed Schwarzschild metric even in strong gravity regime. finally we solved the master equations numerically by setting the initial value conditions using the analytical answers of the strong gravity region. In figures (1) and (2) we plotted the analytical and numerical solutions of the components of the metric, a and b, versus radius in two weak and strong gravity region. It is seen

that in the strong region (the close up part) the relevant analytical answer and the numerical solution are agree together while the analytical weak field approximation solution deviates from the numerical solution. The close up part of figures show that with increasing the radius and going to the weak filed region, the analytical solutions of strong filed approximation and numerical answers get separated from each other, and at last in the weak field region, i.e. $x \gg 1$, the analytical weak field answers coincide with the numerical solution, while the answers for the strong gravity region has a grate deviation from the numerical results in this region.

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